Ch 1. Empirical Approaches to Risk Metrics and Hedging

- **DV01** is the P&L sensitivity (dollar amount) to the underlying risk exposure by basis point.

- Example: A trader bets that the inflation-induced spread will increase

Direction	Bond	Yield %	DV01
Long	TIPS	1.237	0.081
Short	Treasury (Tsry)	3.275	0.067

- <u>DV01-neutral Hedge</u>

$$F_R \text{ X } \frac{0.081}{100} = 100\text{mm X} \frac{0.067}{100}$$

$F_R = 82.7\text{mm}$ every 82.7mm TIPS will be hedged by 100mm Tsry

- <u>Regression Hedge</u>

$$F_R \text{ X } DV01_R = F_N \text{ X } DV01_N \text{ X } \beta$$

When Change in Real Yield (β) is 1.0189, then

$$F_R \text{ X } \frac{0.081}{100} = 100\text{mm X} \frac{0.067}{100} \text{ X } 1.0189 \qquad F_R = 84.3\text{mm}$$

Pitfall of regression hedges is the **instability of an estimated regression coefficient**.

- <u>Two-variable regression based Hedge</u>
  - ◆ Example: due to the illiquidity, 20-year swap needs to be hedged by 10-year and 30-year.

$$F_{10} = -F_{20} \text{ X} \frac{DV01_{20}}{DV01_{10}} \text{ X } \beta_{10}$$

$$F_{30} = -F_{20} \text{ X} \frac{DV01_{20}}{DV01_{30}} \text{ X } \beta_{30}$$

Ch 2. The Science of Term Structure Models

- Constructing the binomial interest rate tree (one underlying rule)
  - The values for on-the-run issues generated using an interest rate tree should prohibit arbitrage free.
  - The value of an on-the-run issue produced by the interest rate tree must equal its market price.

- In order to equate the discounted value using a binomial tree and the market price, we need to use what is known as **risk-neutral probabilities**. Any difference between the risk-neutral and true probabilities is referred to as the **interest rate drift**.

- There are two ways to compute bond and bond derivative values using a binomial model. (**risk-neutral pricing**)
  - *Adjust the interest rates*

  - *Adjust the probabilities*

- Backward-induction methodology with a binomial model requires discounting of the cash flows that occur at each node in an interest rate tree (bond value plus coupon payment) backward to the root of the tree. For bonds with one or more embedded options, the bond value that must be discounted at each node depends on whether the embedded option will be exercised.

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Ex1: Assume a European call option with two years to expiration and a strike price of \$100. The underlying is a 7%, annual-coupon bond with three years to maturity. Assume the risk-neutral probability of an up move is 0.76 in year 1 and 0.6 in year 2.

Ch 3. The Evolution of Short Rates and the Shape of the Term Structure

- Spot or forward rates are determined by expectations of futures short-term rates, the volatility of short-term rate, and an interest rate risk premium.

- **Spot-forward conversion:** Forecast can be useful in describing the sharp and level of the term structure over short-term horizons and the level of rate at very long horizons.

  - Assume $S_1 = 10\%$, $_1f_2 = 12\%$, $_2f_3 = 14\%$

    $\Rightarrow S_1 = 10\%$, $S_2 = 10.995\%$, $S_3 = 11.998\%$

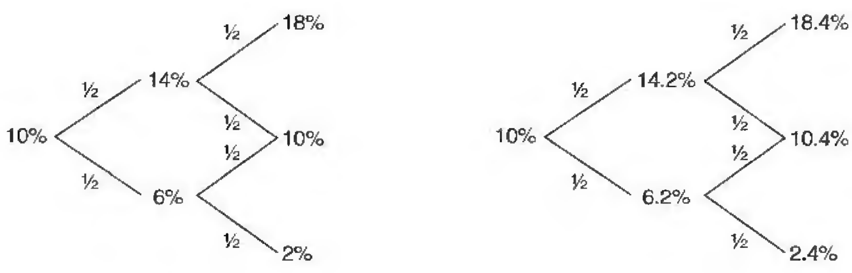
- **Jensen's inequality**

  - Because the pricing function of a zero-coupon, $\frac{1}{1+r}$,is convex, then

$$E\left[\frac{1}{1+r}\right] > \frac{1}{E[1+r]} = \frac{1}{1+E[r]} \quad (*)$$

  - The value of convexity is measured by the distance between the rates assuming no volatility and the rates assuming volatility.
    - Increases by volatility.
    - Increases with maturity

- Any risk-averse investor would prefer an investment with certain return of 10% to an investment with risky return that average 10%.



(a) (b)

  - Convexity increases with maturity and volatility.

- *Risk premium* tends to dominate in the short end while *convexity* tends to dominate in the long end.

Ch 4. The Art of Term Structure Models: Drift

- **General Equation: Rate Change = Drift +/- Volatility**

- ***Model 1: Normally distributed rates and no drift (normal or Gaussian model)***
  - ◆ The continuously compounded, instantaneous rate r_t is assumed to evolve according to the following equation:

$$dr = \sigma dw \quad (*)$$

dr : the change in the rate over a small time interval
σ : the annual basis-point volatility of rate change
$dw = \varepsilon\sqrt{dt}$: the Wiener process, $\varepsilon \sim N(0,1)$
$dw \sim N(0, dt)$
$dr \sim N(0, \sigma^2 dt)$

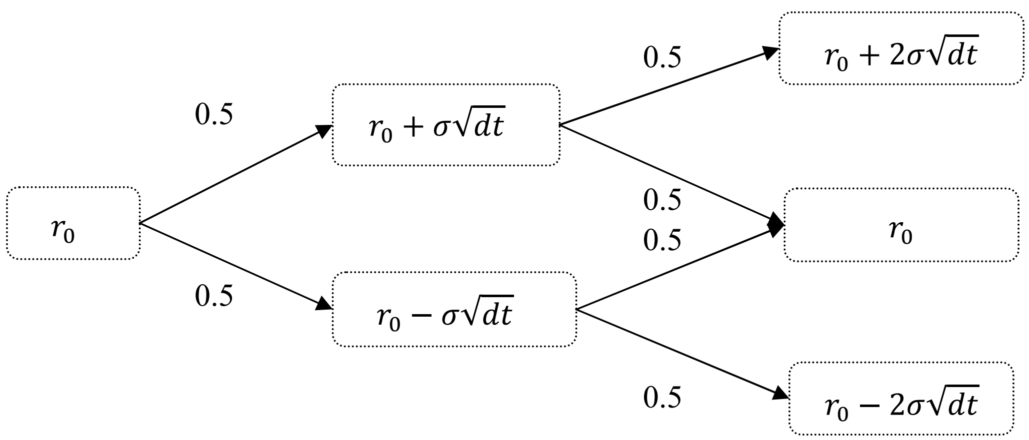
Ex1. Assume $r_0 = 6.18\%,\ \sigma = 113bp,\ dt = \frac{1}{12}$

A month passes and random variable dw, with its zero mean and its standard deviation of $\sqrt{\frac{1}{12}}$ or 0.2887, happen to take on a value of 0.15.

$$dr = 1.13\% \times 0.15 = 0.17\%$$

Since the short-term rate started at 6.18%, the short-term rate after a month is 6.35%.

- ◆ A rate tree is used to approximate the process *



Ch 5. The Art of Term Structure Models: Volatility and Distribution

■ ***Time-dependent volatility Model***

◆ Just as a *time-dependent drift* may be used to fit many bond or swap rate, a *time-dependent volatility function* may be used to fit many option prices.

$$dr = \lambda(t)dt + \sigma(t)dw$$

◆ **Model 3**

$$dr = \lambda(t)dt + \sigma e^{-\alpha t}dw$$

- The volatility of short rate starts at the constant σ and then exponentially declines to zero.
- The standard derivation rises rapidly with horizon at first but then rises more slowly.

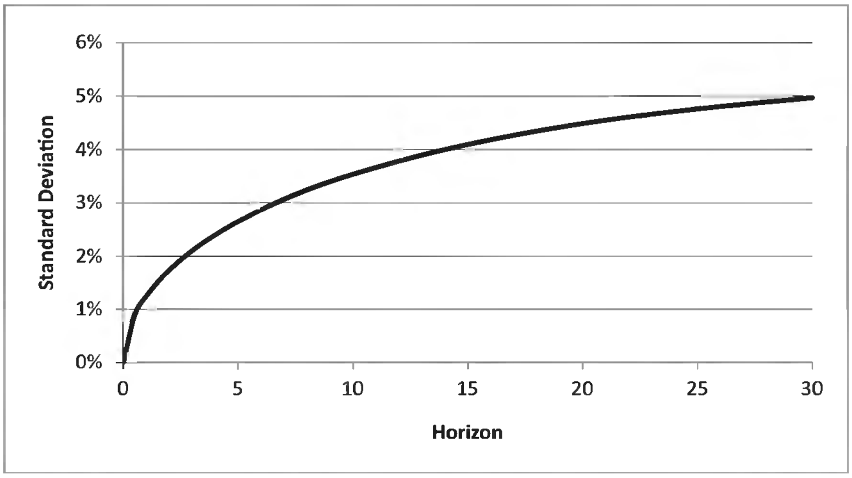


FIGURE 10.1 Standard Deviation of Terminal Distributions of Short Rates in Model 3

■ ***Cox*-Ingersoll-Ross (CIR) *Model***

◆ Volatility as a function of the short rate

$$dr = k(\theta - r)dt + \sigma\sqrt{r}dw$$

$dw = \varepsilon\sqrt{dt}$: the Wiener process, $\varepsilon \sim N(0,1)$
$dw \sim N(0, dt)$
$dr \sim N(0, \sigma^2 r dt)$

- The annual basis-point volatility equal $\sigma\sqrt{r}$, is proportional to the square root of the short rate and increases with the level of the short rate.